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| **Topic/Skill**  | **Definition/Tips** | **Example****Topic: Equations and Formulae**  |
| 1. Solve | To find the **answer**/value of something**Use inverse operations** on both sides of the equation (balancing method) until you find the value for the letter. | Solve $2x-3=7$Add 3 on both sides$$2x=10$$Divide by 2 on both sides$$x=5$$ |
| 2. Inverse | **Opposite** | The inverse of addition is subtraction.The inverse of multiplication is division. |
| 3. Rearranging Formulae | **Use inverse operations** on both sides of the formula (balancing method) until you find the expression for the letter. | Make x the subject of $y=\frac{2x-1}{z}$Multiply both sides by z$$yz=2x-1$$Add 1 to both sides$$yz+1=2x$$Divide by 2 on both sides$$\frac{yz+1}{2}=x$$We now have x as the subject. |
| 4. Writing Formulae | **Substitute letters for words** in the question. | Bob charges £3 per window and a £5 call out charge.$$C=3N+5$$Where N=number of windows and C=cost |
| 5. Substitution | **Replace letters with numbers**.Be careful of $5x^{2}$. You need to square first, then multiply by 5. | $a=3, b=2 and c=5.$ Find:1. $2a=2×3=6$ 2. $3a-2b=3×3-2×2=5$3. $7b^{2}-5=7×2^{2}-5=23$ |

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| **Topic/Skill**  | **Definition/Tips** | **Example****Topic: Solving Quadratics by Factorising**  |
| 1. Quadratic | A quadratic expression is of the form$$ax^{2}+bx+c$$where $a, b$ and $c$ are numbers, $a\ne 0$ | Examples of quadratic expressions:$$x^{2}$$$$8x^{2}-3x+7$$Examples of non-quadratic expressions:$$2x^{3}-5x^{2}$$$$9x-1$$ |
| 2. Factorising Quadratics | When a quadratic expression is in the form $x^{2}+bx+c$ find the two numbers that **add to give b** and **multiply to give c**. | $$x^{2}+7x+10=(x+5)(x+2)$$(because 5 and 2 add to give 7 and multiply to give 10)$$x^{2}+2x-8=(x+4)(x-2)$$(because +4 and -2 add to give +2 and multiply to give -8) |
| 3. Difference of Two Squares | An expression of the form $a^{2}-b^{2}$ can be factorised to give $(a+b)(a-b)$ | $$x^{2}-25=(x+5)(x-5)$$$$16x^{2}-81=(4x+9)(4x-9)$$ |
| 4. Solving Quadratics $(ax^{2}=b)$ | Isolate the $x^{2}$ term and square root both sides.Remember there will be a **positive and a negative solution**. | $$2x^{2}=98$$$$x^{2}=49$$$$x=\pm 7$$ |
| 5. Solving Quadratics $(ax^{2}+bx=0)$ | **Factorise** and then **solve = 0**. | $$x^{2}-3x=0$$$$x\left(x-3\right)=0$$$$x=0 or x=3$$ |
| 6. Solving Quadratics by Factorising $\left(a=1\right)$  | **Factorise** the quadratic in the usual way.**Solve = 0** Make sure the equation = 0 before factorising. | Solve $x^{2}+3x-10=0$Factorise: $\left(x+5\right)\left(x-2\right)=0$$$x=-5 or x=2$$ |
| 7. Factorising Quadratics when $a\ne 1$ | When a quadratic is in the form$$ax^{2}+bx+c$$1. Multiply a by c = ac2. Find two numbers that add to give b and multiply to give ac.3. Re-write the quadratic, replacing $bx$ with the two numbers you found.4. Factorise in pairs – you should get the same bracket twice5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets. | Factorise $6x^{2}+5x-4$1. $6×-4=-24$2. Two numbers that add to give +5 and multiply to give -24 are +8 and -33. $6x^{2}+8x-3x-4$4. Factorise in pairs: $$2x\left(3x+4\right)-1(3x+4)$$5. Answer = $(3x+4)(2x-1)$ |
| 8. Solving Quadratics by Factorising $\left(a\ne 1\right)$  | **Factorise** the quadratic in the usual way.**Solve = 0** Make sure the equation = 0 before factorising. | Solve $2x^{2}+7x-4=0$Factorise: $\left(2x-1\right)\left(x+4\right)=0$$$x=\frac{1}{2} or x=-4$$ |

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| **Topic/Skill**  | **Definition/Tips** | **Example****Topic: Further Quadratics**  |
| 1. Quadratic | A quadratic expression is of the form$$ax^{2}+bx+c$$where $a, b$ and $c$ are numbers, $a\ne 0$ | Examples of quadratic expressions:$$x^{2}$$$$8x^{2}-3x+7$$Examples of non-quadratic expressions:$$2x^{3}-5x^{2}$$$$9x-1$$ |
| 2. Factorising Quadratics | When a quadratic expression is in the form $x^{2}+bx+c$ find the two numbers that **add to give b** and **multiply to give c**. | $$x^{2}+7x+10=(x+5)(x+2)$$(because 5 and 2 add to give 7 and multiply to give 10)$$x^{2}+2x-8=(x+4)(x-2)$$(because +4 and -2 add to give +2 and multiply to give -8) |
| 3. Difference of Two Squares | An expression of the form $a^{2}-b^{2}$ can be factorised to give $(a+b)(a-b)$ | $$x^{2}-25=(x+5)(x-5)$$$$16x^{2}-81=(4x+9)(4x-9)$$ |
| 4. Solving Quadratics $(ax^{2}=b)$ | Isolate the $x^{2}$ term and square root both sides.Remember there will be a **positive and a negative solution**. | $$2x^{2}=98$$$$x^{2}=49$$$$x=\pm 7$$ |
| 5. Solving Quadratics $(ax^{2}+bx=0)$ | **Factorise** and then **solve = 0**. | $$x^{2}-3x=0$$$$x\left(x-3\right)=0$$$$x=0 or x=3$$ |
| 6. Solving Quadratics by Factorising $\left(a=1\right)$  | **Factorise** the quadratic in the usual way.**Solve = 0** Make sure the equation = 0 before factorising. | Solve $x^{2}+3x-10=0$Factorise: $\left(x+5\right)\left(x-2\right)=0$$$x=-5 or x=2$$ |
| 7. Quadratic Graph | A ‘**U-shaped**’ curve called a **parabola**.The equation is of the form$y=ax^{2}+bx+c$, where $a$, $b$ and $c$ are numbers, $a\ne 0$. If $a<0$**,** the parabola is **upside down**. | Image result for quadratic graph definition math |
| 8. Roots of a Quadratic  | A root is a **solution**.The roots of a quadratic are the $x$**-intercepts of the quadratic graph**. | Image result |
| 9. Turning Point of a Quadratic | A turning point is the **point where a quadratic turns**.On a **positive parabola**, the turning point is called a **minimum**.On a **negative parabola**, the turning point is called a **maximum**. | Minimum turning pointMaximum turning point |
| 10. Factorising Quadratics when $a\ne 1$ | When a quadratic is in the form$$ax^{2}+bx+c$$1. Multiply a by c = ac2. Find two numbers that add to give b and multiply to give ac.3. Re-write the quadratic, replacing $bx$ with the two numbers you found.4. Factorise in pairs – you should get the same bracket twice5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets. | Factorise $6x^{2}+5x-4$1. $6×-4=-24$2. Two numbers that add to give +5 and multiply to give -24 are +8 and -33. $6x^{2}+8x-3x-4$4. Factorise in pairs: $$2x\left(3x+4\right)-1(3x+4)$$5. Answer = $(3x+4)(2x-1)$ |
| 11. Solving Quadratics by Factorising $\left(a\ne 1\right)$  | **Factorise** the quadratic in the usual way.**Solve = 0** Make sure the equation = 0 before factorising. | Solve $2x^{2}+7x-4=0$Factorise: $\left(2x-1\right)\left(x+4\right)=0$$$x=\frac{1}{2} or x=-4$$ |
| 12. Completing the Square (when $a=1)$ | A quadratic in the form $x^{2}+bx+c$ can be written in the form $(x+p)^{2}+q$1. Write a set of brackets with $x$ in and **half** the value of $b.$2. Square the bracket.3. Subtract $\left(\frac{b}{2}\right)^{2}$and add $c.$4. Simplify the expression.You can **use the completing the square form** to help **find the maximum or minimum** of quadratic graph. | Complete the square of $$y=x^{2}-6x+2$$Answer:$$(x-3)^{2}-3^{2}+2$$$$=(x-3)^{2}-7$$The minimum value of this expression occurs when $(x-3)^{2}=0$, which occurs when $x=3$When $x=3$, $y=0-7=-7$Minimum point = $(3,-7)$ |
| 13. Completing the Square (when $a\ne 1)$ | A quadratic in the form $ax^{2}+bx+c$ can be written in the form **p**$(x+q)^{2}+r$Use the same method as above, but factorise out $a$ at the start. | Complete the square of $$4x^{2}+8x-3$$Answer:$$4\left[x^{2}+2x\right]-3$$$$=4\left[\left(x+1\right)^{2}-1^{2}\right]-3$$$$=4(x+1)^{2}-4-3$$$$=4(x+1)^{2}-7$$ |
| 14. Solving Quadratics by Completing the Square | **Complete the square** in the usual way and **use inverse operations to solve**. | Solve $x^{2}+8x+1=0$Answer:$$\left(x+4\right)^{2}-4^{2}+1=0$$$$\left(x+4\right)^{2}-15=0$$$$\left(x+4\right)^{2}=15$$$$\left(x+4\right)=\pm \sqrt{15}$$$$x=-4\pm \sqrt{15}$$ |
| 15. Solving Quadratics using the Quadratic Formula | A quadratic in the form $ax^{2}+bx+c=0$ can be solved using the formula:$$x=\frac{-b\pm \sqrt{b^{2}-4ac}}{2a}$$Use the formula if the quadratic does not factorise easily. | Solve $3x^{2}+x-5=0$Answer:$a=3, b=1, c=-5$ $$x=\frac{-1\pm \sqrt{1^{2}-4×3×-5}}{2×3}$$$$x=\frac{-1\pm \sqrt{61}}{6}$$$$x=1.14 or-1.47 (2 d.p.)$$ |

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| **Topic/Skill**  | **Definition/Tips** | **Example****Topic: Inequalities**  |
| 1. Inequality  | An inequality says that two values are **not equal**.$a\ne b$ means that a is not equal to b. | $$7\ne 3$$$$x\ne 0$$ |
| 2. Inequality symbols | $x>2$ means **x is greater than 2**$x<3$ means **x is less than 3**$x\geq 1$ means **x is greater than or equal to 1**$x\leq 6 $means **x is less than or equal to 6** | State the integers that satisfy $$-2<x\leq 4.$$-1, 0, 1, 2, 3, 4 |
| 3. Inequalities on a Number Line | Inequalities can be shown on a number line.**Open circles** are used for numbers that are **less than or greater than** $(<or>)$**Closed circles** are used for numbers that are **less than or equal or greater than or equal** $(\leq or\geq )$  | $x\geq 0$$x<2$$-5\leq x<4$ |
| 4. Graphical Inequalities | Inequalities can be represented on a coordinate grid.If the inequality is **strict** ($x>2$) then use a **dotted line**.If the inequality is **not strict** ($x\leq 6$) then use a **solid line**.**Shade** the **region** which satisfies all the inequalities. | Shade the region that satisfies:$$y>2x, x>1 and y\leq 3$$ |
| 5. Quadratic Inequalities | **Sketch the quadratic graph** of the inequality.If the expression is $>or\geq $ then the answer will be **above the x-axis**.If the expression is $<or\leq $ then the answer will be **below the x-axis**.Look carefully at the inequality symbol in the question.Look carefully if the quadratic is a **positive or negative parabola**. | Solve the inequality $x^{2}-x-12<0$Sketch the quadratic:The required region is below the x-axis, so the final answer is:$$-3<x<4$$If the question had been $>0$, the answer would have been:$$x<-3 or x>4$$ |
| 6. Set Notation | A **set** is a **collection of things**, usually numbers, denoted with brackets $\{ \}$$\left\{x \right| x\geq 7\}$ means ‘the set of all x’s, such that x is greater than or equal to 7’The ‘x’ can be replaced by any letter.Some people use ‘:’ instead of ‘|’ | $\{3, 6, 9\}$ is a set.$$\{x : -2\leq x<5\}$$ |

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| **Topic/Skill**  | **Definition/Tips** | **Example****Topic: Simultaneous Equations**  |
| 1. Simultaneous Equations | A set of **two or more equations**, each involving **two or more variables** (letters).The **solutions** to simultaneous equations **satisfy both**/all of the **equations**. | $$2x+y=7$$$$3x-y=8$$$$x=3$$$$y=1$$ |
| 2. Variable | A **symbol**, usually a **letter**, which **represents a number** which is usually unknown.  | In the equation $x+2=5$, $x$ is the variable. |
| 3. Coefficient | A **number** used to **multiply** a **variable**.It is the number that comes before/in front of a letter. | 6z6 is the coefficientz is the variable |
| 4. Solving Simultaneous Equations (by Elimination) | 1. **Balance** the **coefficients** of one of the variables.2. **Eliminate** this variable by adding or subtracting the equations (**Same Sign Subtract, Different Sign Add**)3. **Solve** the linear equation you get using the other variable.4. **Substitute** the value you found back into one of the previous equations.5. **Solve** the equation you get.6. **Check** that the two values you get satisfy both of the original equations. | $$5x+2y=9$$$$10x+3y=16$$Multiply the first equation by 2.$$10x+4y=18$$$$10x+3y=16$$Same Sign Subtract (+10x on both)$$y=2$$Substitute $y=2$ in to equation.$$5x+2×2=9$$$$5x+4=9$$$$5x=5$$$$x=1$$Solution: $x=1, y=2$ |
| 5. Solving Simultaneous Equations (by Substitution) | 1. **Rearrange** one of the equations into the form $y=...$ or $x=...$2. **Substitute** the right-hand side of the rearranged equation into the other equation.3. Expand and **solve** this equation.4. **Substitute** the value into the $y=...$ or $x=...$ equation.5.  **Check** that the two values you get satisfy both of the original equations. | $$y-2x=3$$$$3x+4y=1$$Rearrange: $y-2x=3\rightarrow y=2x+3$Substitute: $3x+4\left(2x+3\right)=1$Solve: $3x+8x+12=1$$$11x=-11$$$$x=-1$$Substitute: $y=2×-1+3$$$y=1$$Solution: $x=-1, y=1$ |

**Knowledge Organiser**