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| **Topic/Skill**  | **Definition/Tips** | **Example****Topic: Algebra**  |
| 1. Expression | A mathematical statement written using **symbols**, **numbers** or **letters**, | 3x + 2 or 5y2 |
| 2. Equation | A statement showing that **two expressions are equal** | 2y – 17 = 15 |
| 3. Identity | An equation that is **true for all values** of the variablesAn identity uses the symbol: $≡$ | *2x ≡ x+x* |
| 4. Formula | Shows the **relationship** between **two or more variables** | Area of a rectangle = length x width or A= LxW |
| 5. Simplifying Expressions | **Collect ‘like terms’.** Be careful with negatives. $x^{2}$ and $x$ are not like terms. | $$2x+3y+4x-5y+3=6x-2y+3$$$$3x+4-x^{2}+2x-1=5x-x^{2}+3$$ |
| 6. $x $times $x$ | The answer is $x^{2}$ not $2x$. | Squaring is multiplying by itself, not by 2. |
| 7. $p×p×p$  | The answer is $p^{3}$ not $3p$ | If p=2, then $p^{3}$=2x2x2=8, not 2x3=6 |
| 8. $p+p+p$  | The answer is 3p not $p^{3}$ | If p=2, then 2+2+2=6, not $2^{3}=8$ |
| 9. Expand | To expand a bracket, **multiply** each term **in the bracket** by the expression **outside** the bracket. | $$3\left(m+7\right)=3x+21$$ |
| 10. Factorise | The **reverse** of **expanding**.Factorising is writing an expression as a product of terms by ‘**taking out’ a common factor**. | $6x-15=3(2x-5)$, where 3 is the common factor. |

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| **Topic/Skill**  | **Definition/Tips** | **Example****Topic: Solving Quadratics by Factorising**  |
| 1. Quadratic | A quadratic expression is of the form$$ax^{2}+bx+c$$where $a, b$ and $c$ are numbers, $a\ne 0$ | Examples of quadratic expressions:$$x^{2}$$$$8x^{2}-3x+7$$Examples of non-quadratic expressions:$$2x^{3}-5x^{2}$$$$9x-1$$ |
| 2. Factorising Quadratics | When a quadratic expression is in the form $x^{2}+bx+c$ find the two numbers that **add to give b** and **multiply to give c**. | $$x^{2}+7x+10=(x+5)(x+2)$$(because 5 and 2 add to give 7 and multiply to give 10)$$x^{2}+2x-8=(x+4)(x-2)$$(because +4 and -2 add to give +2 and multiply to give -8) |
| 3. Difference of Two Squares | An expression of the form $a^{2}-b^{2}$ can be factorised to give $(a+b)(a-b)$ | $$x^{2}-25=(x+5)(x-5)$$$$16x^{2}-81=(4x+9)(4x-9)$$ |
| 4. Solving Quadratics $(ax^{2}=b)$ | Isolate the $x^{2}$ term and square root both sides.Remember there will be a **positive and a negative solution**. | $$2x^{2}=98$$$$x^{2}=49$$$$x=\pm 7$$ |
| 5. Solving Quadratics $(ax^{2}+bx=0)$ | **Factorise** and then **solve = 0**. | $$x^{2}-3x=0$$$$x\left(x-3\right)=0$$$$x=0 or x=3$$ |
| 6. Solving Quadratics by Factorising $\left(a=1\right)$  | **Factorise** the quadratic in the usual way.**Solve = 0** Make sure the equation = 0 before factorising. | Solve $x^{2}+3x-10=0$Factorise: $\left(x+5\right)\left(x-2\right)=0$$$x=-5 or x=2$$ |
| 7. Factorising Quadratics when $a\ne 1$ | When a quadratic is in the form$$ax^{2}+bx+c$$1. Multiply a by c = ac2. Find two numbers that add to give b and multiply to give ac.3. Re-write the quadratic, replacing $bx$ with the two numbers you found.4. Factorise in pairs – you should get the same bracket twice5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets. | Factorise $6x^{2}+5x-4$1. $6×-4=-24$2. Two numbers that add to give +5 and multiply to give -24 are +8 and -33. $6x^{2}+8x-3x-4$4. Factorise in pairs: $$2x\left(3x+4\right)-1(3x+4)$$5. Answer = $(3x+4)(2x-1)$ |
| 8. Solving Quadratics by Factorising $\left(a\ne 1\right)$  | **Factorise** the quadratic in the usual way.**Solve = 0** Make sure the equation = 0 before factorising. | Solve $2x^{2}+7x-4=0$Factorise: $\left(2x-1\right)\left(x+4\right)=0$$$x=\frac{1}{2} or x=-4$$ |

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| **Topic/Skill**  | **Definition/Tips** | **Example****Topic: Further Quadratics**  |
| 1. Quadratic | A quadratic expression is of the form$$ax^{2}+bx+c$$where $a, b$ and $c$ are numbers, $a\ne 0$ | Examples of quadratic expressions:$$x^{2}$$$$8x^{2}-3x+7$$Examples of non-quadratic expressions:$$2x^{3}-5x^{2}$$$$9x-1$$ |
| 2. Factorising Quadratics | When a quadratic expression is in the form $x^{2}+bx+c$ find the two numbers that **add to give b** and **multiply to give c**. | $$x^{2}+7x+10=(x+5)(x+2)$$(because 5 and 2 add to give 7 and multiply to give 10)$$x^{2}+2x-8=(x+4)(x-2)$$(because +4 and -2 add to give +2 and multiply to give -8) |
| 3. Difference of Two Squares | An expression of the form $a^{2}-b^{2}$ can be factorised to give $(a+b)(a-b)$ | $$x^{2}-25=(x+5)(x-5)$$$$16x^{2}-81=(4x+9)(4x-9)$$ |
| 4. Solving Quadratics $(ax^{2}=b)$ | Isolate the $x^{2}$ term and square root both sides.Remember there will be a **positive and a negative solution**. | $$2x^{2}=98$$$$x^{2}=49$$$$x=\pm 7$$ |
| 5. Solving Quadratics $(ax^{2}+bx=0)$ | **Factorise** and then **solve = 0**. | $$x^{2}-3x=0$$$$x\left(x-3\right)=0$$$$x=0 or x=3$$ |
| 6. Solving Quadratics by Factorising $\left(a=1\right)$  | **Factorise** the quadratic in the usual way.**Solve = 0** Make sure the equation = 0 before factorising. | Solve $x^{2}+3x-10=0$Factorise: $\left(x+5\right)\left(x-2\right)=0$$$x=-5 or x=2$$ |
| 7. Quadratic Graph | A ‘**U-shaped**’ curve called a **parabola**.The equation is of the form$y=ax^{2}+bx+c$, where $a$, $b$ and $c$ are numbers, $a\ne 0$. If $a<0$**,** the parabola is **upside down**. | Image result for quadratic graph definition math |
| 8. Roots of a Quadratic  | A root is a **solution**.The roots of a quadratic are the $x$**-intercepts of the quadratic graph**. | Image result |
| 9. Turning Point of a Quadratic | A turning point is the **point where a quadratic turns**.On a **positive parabola**, the turning point is called a **minimum**.On a **negative parabola**, the turning point is called a **maximum**. | Minimum turning pointMaximum turning point |
| 10. Factorising Quadratics when $a\ne 1$ | When a quadratic is in the form$$ax^{2}+bx+c$$1. Multiply a by c = ac2. Find two numbers that add to give b and multiply to give ac.3. Re-write the quadratic, replacing $bx$ with the two numbers you found.4. Factorise in pairs – you should get the same bracket twice5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets. | Factorise $6x^{2}+5x-4$1. $6×-4=-24$2. Two numbers that add to give +5 and multiply to give -24 are +8 and -33. $6x^{2}+8x-3x-4$4. Factorise in pairs: $$2x\left(3x+4\right)-1(3x+4)$$5. Answer = $(3x+4)(2x-1)$ |
| 11. Solving Quadratics by Factorising $\left(a\ne 1\right)$  | **Factorise** the quadratic in the usual way.**Solve = 0** Make sure the equation = 0 before factorising. | Solve $2x^{2}+7x-4=0$Factorise: $\left(2x-1\right)\left(x+4\right)=0$$$x=\frac{1}{2} or x=-4$$ |
| 12. Completing the Square (when $a=1)$ | A quadratic in the form $x^{2}+bx+c$ can be written in the form $(x+p)^{2}+q$1. Write a set of brackets with $x$ in and **half** the value of $b.$2. Square the bracket.3. Subtract $\left(\frac{b}{2}\right)^{2}$and add $c.$4. Simplify the expression.You can **use the completing the square form** to help **find the maximum or minimum** of quadratic graph. | Complete the square of $$y=x^{2}-6x+2$$Answer:$$(x-3)^{2}-3^{2}+2$$$$=(x-3)^{2}-7$$The minimum value of this expression occurs when $(x-3)^{2}=0$, which occurs when $x=3$When $x=3$, $y=0-7=-7$Minimum point = $(3,-7)$ |
| 13. Completing the Square (when $a\ne 1)$ | A quadratic in the form $ax^{2}+bx+c$ can be written in the form **p**$(x+q)^{2}+r$Use the same method as above, but factorise out $a$ at the start. | Complete the square of $$4x^{2}+8x-3$$Answer:$$4\left[x^{2}+2x\right]-3$$$$=4\left[\left(x+1\right)^{2}-1^{2}\right]-3$$$$=4(x+1)^{2}-4-3$$$$=4(x+1)^{2}-7$$ |
| 14. Solving Quadratics by Completing the Square | **Complete the square** in the usual way and **use inverse operations to solve**. | Solve $x^{2}+8x+1=0$Answer:$$\left(x+4\right)^{2}-4^{2}+1=0$$$$\left(x+4\right)^{2}-15=0$$$$\left(x+4\right)^{2}=15$$$$\left(x+4\right)=\pm \sqrt{15}$$$$x=-4\pm \sqrt{15}$$ |
| 15. Solving Quadratics using the Quadratic Formula | A quadratic in the form $ax^{2}+bx+c=0$ can be solved using the formula:$$x=\frac{-b\pm \sqrt{b^{2}-4ac}}{2a}$$Use the formula if the quadratic does not factorise easily. | Solve $3x^{2}+x-5=0$Answer:$a=3, b=1, c=-5$ $$x=\frac{-1\pm \sqrt{1^{2}-4×3×-5}}{2×3}$$$$x=\frac{-1\pm \sqrt{61}}{6}$$$$x=1.14 or-1.47 (2 d.p.)$$ |

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| **Topic/Skill**  | **Definition/Tips** | **Example****Topic: Equations and Formulae**  |
| 1. Solve | To find the **answer**/value of something**Use inverse operations** on both sides of the equation (balancing method) until you find the value for the letter. | Solve $2x-3=7$Add 3 on both sides$$2x=10$$Divide by 2 on both sides$$x=5$$ |
| 2. Inverse | **Opposite** | The inverse of addition is subtraction.The inverse of multiplication is division. |
| 3. Rearranging Formulae | **Use inverse operations** on both sides of the formula (balancing method) until you find the expression for the letter. | Make x the subject of $y=\frac{2x-1}{z}$Multiply both sides by z$$yz=2x-1$$Add 1 to both sides$$yz+1=2x$$Divide by 2 on both sides$$\frac{yz+1}{2}=x$$We now have x as the subject. |
| 4. Writing Formulae | **Substitute letters for words** in the question. | Bob charges £3 per window and a £5 call out charge.$$C=3N+5$$Where N=number of windows and C=cost |
| 5. Substitution | **Replace letters with numbers**.Be careful of $5x^{2}$. You need to square first, then multiply by 5. | $a=3, b=2 and c=5.$ Find:1. $2a=2×3=6$ 2. $3a-2b=3×3-2×2=5$3. $7b^{2}-5=7×2^{2}-5=23$ |

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| **Topic/Skill**  | **Definition/Tips** | **Example****Topic: Sequences**  |
| 1. Linear Sequence | A number pattern with a **common difference**. | 2, 5, 8, 11… is a linear sequence |
| 2. Term | **Each value** in a sequence is called a term. | In the sequence 2, 5, 8, 11…, 8 is the third term of the sequence. |
| 3. Term-to-term rule | A rule which allows you to **find the next term** in a sequence if you **know the previous term**. | First term is 2. Term-to-term rule is ‘add 3’Sequence is: 2, 5, 8, 11… |
| 4. nth term | A rule which allows you to **calculate the term** that is in the **nth position** of the sequence.Also known as the ‘position-to-term’ rule.**n** refers to the **position** of a term in a sequence. | nth term is $3n-1$The 100th term is $3×100-1=299$ |
| 5. Finding the nth term of a linear sequence | 1. Find the **difference**.2. **Multiply that by** $n.$3. Substitute $n=1$ to **find out what number you need to add or subtract to get the first number in the sequence**. | Find the nth term of: 3, 7, 11, 15…1. Difference is +42. Start with $4n$3. $4×1=4$, so we need to subtract 1 to get 3.nth term = $4n-1$ |
| 6. Fibonacci type sequences | A sequence where the next number is found by **adding up the previous two terms**  | The Fibonacci sequence is:$$1,1,2,3,5,8,13,21,34…$$An example of a Fibonacci-type sequence is:$$4, 7, 11, 18, 29…$$ |
| 7. Geometric Sequence | A sequence of numbers where each term is found by **multiplying the previous one** by a number called the **common ratio, r**. | An example of a geometric sequence is:$$2, 10, 50, 250…$$The common ratio is 5Another example of a geometric sequence is:$$81, -27, 9,-3, 1… $$The common ratio is $-\frac{1}{3}$ |
| 8. Quadratic Sequence | A sequence of numbers where the **second difference is constant**.A quadratic sequence will have a $n^{2}$ term. | quadratic sequence: 2, 6, 12, 20, 30, 42 |
| 9. nth term of a geometric sequence | $$ar^{n-1}$$where $a$ is the first term and $r$ is the common ratio | The nth term of $2, 10, 50, 250….$ Is$$2×5^{n-1}$$ |
| 10. nth term of a quadratic sequence | 1. Find the first and second differences.2. Halve the second difference and multiply this by $n^{2}$.3. Substitute $n=1,2,3,4…$ into your expression so far.4. Subtract this set of numbers from the corresponding terms in the sequence from the question.5. Find the nth term of this set of numbers.6. Combine the nth terms to find the overall nth term of the quadratic sequence.Substitute values in to check your nth term works for the sequence. | Find the nth term of: 4, 7, 14, 25, 40..Answer:Second difference = +4 🡪 nth term = $2n^{2}$Sequence: 4, 7, 14, 25, 40$2n^{2}$ 2, 8, 18, 32, 50Difference: 2, -1, -4, -7, -10Nth term of this set of numbers is $-3n+5$Overall nth term: $2n^{2}-3n+5$ |
| 11. Triangular numbers | The sequence which comes from a pattern of dots that form a triangle.$$1, 3, 6, 10, 15, 21…$$ |  |

**Knowledge Organiser**